Short Papers

Noise Parameter Transformations for Three-Terminal Amplifiers

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Abstract —The common-emitter noise parameters and z parameters for a transistor (or any three-terminal amplifier) are used to obtain the corresponding sets of noise parameters for the common-base and common-collector configurations. It is shown that the three configurations must have the same minimum noise measure. A practical example is presented to confirm this invariance.

I. INTRODUCTION

Most amplifier designers are familiar with noise matching, where an amplifier's input network transforms the impedance of the intended source in order to minimize the noise figure rather than maximize the gain. It is also commonly known that a better figure of merit than noise figure is noise measure, $M = (NF - 1)/(1 - 1/G)$, since one can show immediately that the noise measure gives the excess noise figure for a cascade of identical amplifiers. Normally one thinks of a cascade with enough total gain that its noise determines the system noise figure. Minimum noise measure, like minimum noise figure, can generally be realized with an appropriate input matching network. Haus and Adler [1] showed that the minimum noise measure of an amplifier (which can be an isolated transistor) also applies to a new amplifier made by embedding the original amplifier in an arbitrary network of lossless (purely reactive) elements. This explained why lossless feedback can improve an amplifier's noise figure at the expense of its gain (or improve gain at the expense of noise figure). Since one can easily devise such a network to do nothing more than interchange terminals, it also follows that the minimum noise measure of an amplifier is invariant with respect to the circuit configuration: common-emitter, common-base, or common-collector.

In this paper the common-base (CB) and common-collector (CC) noise parameters are derived directly from the common-emitter (CE) noise parameters and z parameters, and a practical example of these transformations is presented to confirm the invariance of the noise measure.

II. COMMON-BASE AND COMMON-COLLECTOR NOISE PARAMETERS

The transformations discussed here apply, of course, to any three-terminal device: FET, vacuum tube, complete amplifier, etc. Bipolar transistor notation is used here only because the terms "common-emitter," etc. are universally familiar and the schematic symbol for the bipolar transistor is unambiguous; the transistor analyzed in Section IV is a HEMT.

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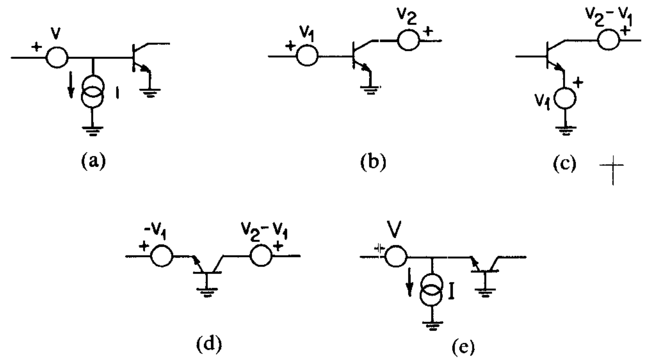

Fig. 1. CE-to-CB transformation.

Fig. 1(a) shows the starting point, the common-emitter configuration with its noise represented by equivalent input sources: a current generator and a voltage generator. These sources and their (complex) correlation coefficient totally characterize the noise of the device. Normally these sources are specified by the following standard set of noise parameters [2]:

$$r = \langle |v|^2 \rangle / 4kT_0 \tag{1}$$

$$g = \langle |i|^2 \rangle / 4kT_0 \tag{2}$$

$$y_c = g_c + jb_c = \langle v^* i \rangle / \langle |v|^2 \rangle. \tag{3}$$

These parameters are known as noise resistance, noise conductance, and correlation admittance [2]. Boltzmann's constant and the reference temperature are denoted by k and T_0. Lowercase letters are used in this paper for common-emitter parameters. The noise sources are spectral densities, i.e., volts and amperes per root Hz. In terms of these parameters, the noise figure of the device is given by [2]

$$F = F_{\text{opt}} + \frac{r}{G_s} |Y_s - (G_{\text{opt}} + jB_{\text{opt}})|^2 \tag{4}$$

where $Y_s = G_s + jB_s$ is the admittance of the source, and

$$G_{\text{opt}}^2 = g_c^2 + \frac{1}{r}(g - r|y_c|^2) \tag{5}$$

$$B_{\text{opt}} = -b_c \tag{6}$$

and

$$F_{\text{opt}} = 1 + 2r(G_{\text{opt}} + g_c). \tag{7}$$

A. Common-Base

Parts (a)–(e) of Fig. 1 show a sequence of transformations that proceed from the CE configuration and end at the CB configuration. The voltage and current noise sources of Fig. 1(a) are easily transformed [2] into noise voltage sources, v_1 and v_2, given by

$$v_1 = v - iz_{11} \tag{8}$$

$$v_2 = -iz_{21} \tag{9}$$

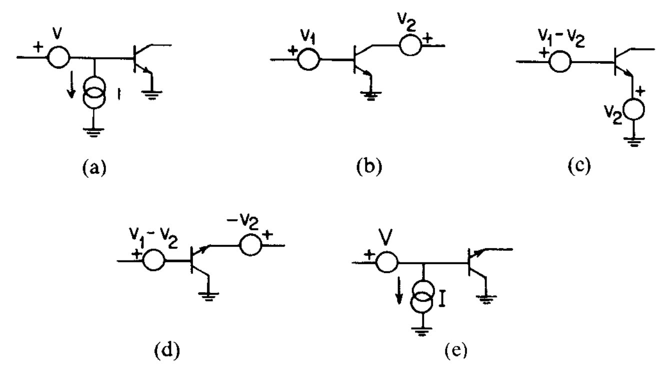

Fig. 2. CE-to-CC transformation.

where the lowercase z denotes CE z parameters. The inverse of the transformation used to proceed from (a) to (b) in Fig. 1 is used to go from (d) to (e). The resulting CB noise sources are given by

$$V = v_1\left[\frac{Z_{11}}{Z_{21}} - 1\right] - v_2\left[\frac{Z_{11}}{Z_{21}}\right] \qquad I = \frac{v_1 - v_2}{Z_{21}} \tag{10}$$

where uppercase letters are used for CB parameters. The z-to-Z conversions are well known and straightforward. Using the definitions in (1) to (3), the CB noise parameters become

$$R = r|C_1|^2 + g|C_2|^2 + 2r\,\mathrm{Re}(y_c C_1 C_2) \tag{11}$$

$$G = \frac{1}{|Z_{21}|^2}\left[r + g|C_3|^2 + 2r\,\mathrm{Re}(y_c C_3)\right] \tag{12}$$

$$Y_c = \frac{1}{RZ_{21}}\left[rC_1^* + gC_3C_2^* + ry_c^*C_2^* + y_c r C_3 C_1^*\right] \tag{13}$$

where

$$C_1 = \frac{Z_{11}^*}{Z_{21}^*} - 1 \tag{14}$$

$$C_2 = z_{11} + (z_{21} - z_{11})\left[\frac{Z_{11}}{Z_{21}}\right] \tag{15}$$

and

$$C_3 = z_{21} - z_{11}. \tag{16}$$

B. Common-Collector

Fig. 2 shows the steps used to derive the common-collector configuration. The resulting CC noise parameters are

$$R = r + g|C_4|^2 - 2r\,\mathrm{Re}(y_c C_4) \tag{17}$$

$$G = g|z_{21}|^2/|Z_{21}|^2 \tag{18}$$

$$Y_c = -\frac{z_{21}}{RZ_{21}}\left[ry_c - gC_4^*\right] \tag{19}$$

where

$$C_4 = z_{11} + z_{21}\left[\frac{Z_{11}}{Z_{21}} - 1\right]. \tag{20}$$

Lowercase letters still indicate common-emitter while uppercase letters now indicate common-collector.

III. Noise Measure

A network that only permutes the three transistor terminals is within the class of lossless embedding networks considered by Haus and Adler. They showed that if the resulting amplifier has $|\mathrm{Gain}| > 1$, its noise measure cannot be less than that of the original amplifier. It follows that the minimum noise measure for the three circuit configurations must be identical. Optimum system performance can be realized with any of the three configurations (or combinations thereof) and appropriate lossless passive circuit elements to produce a high-gain amplifier whose excess noise figure ($NF - 1$) is equal to the noise measure of the basic transistor. The choice of CE, CB, or CC for the input stage need not be made, then, on the basis of noise, but can be made, for example, to lower the input reflection coefficient or for stability or for the lowest Q of the optimum source impedance in the interest of wide-band noise matching. Other means to these ends are lossless feedback and paralleling of devices. In all cases, the minimum noise measure is that of the basic transistor. An amplifier containing more than one device cannot have a minimum overall noise measure better than that of its best device [1].

Note that the gain, G, in the definition of noise measure is the so-called available gain, i.e., the gain obtained with a given source impedance, Z_s, and a conjugately matched load:

$$G = |Z_{21}|^2\,\mathrm{Re}(Z_s)/\mathrm{Re}\left[\left[Z_{22}(Z_{11} + Z_s) - Z_{21}Z_{12}\right]\left[Z_{11}^* + Z_s^*\right]\right]. \tag{21}$$

IV. A Practical Example

The common-base and common-collector noise parameters for a HEMT at 18 GHz were calculated from its common-emitter noise parameters using the equations derived above. Then the minimum noise measure and corresponding source impedance for each of the three configurations were calculated using the eigenvalue formulation of Haus and Adler [1]. The common-emitter noise parameters and S parameters were taken from the manufacturer's data sheet.

The top half of Table I shows the minimum noise figure, F_{opt}, as well as the corresponding source admittance and the resulting available gain and noise measure for each configuration. The bottom half of the table shows the minimum noise measure, M_{min}, its corresponding source admittance, and the resulting available gain and noise figure. Note that the minimum noise measure is indeed the same (638°) for the three configurations. The minimum noise figures, expressed as excess noise temperatures, are 522° for CE, 580° for CB, and 510° for CC. However, if one were to build a 510° CC amplifier, the resulting noise measure would be 1098°. This is an example where matching for minimum noise figure instead of minimum noise measure would seriously degrade system performance. Such examples will ordinarily be found only at the highest useful frequencies for a given device; at lower frequencies all three configurations will produce enough gain to make the minimum excess noise figure and minimum noise measure virtually identical.

Examination of Table I shows that the gain of the CC amplifier is negative when presented with the source impedance corresponding to the minimum noise measure. This indicates that the output impedance is negative and the amplifier will be unstable. Nevertheless, Haus and Adler have shown that, by unilateralizing the amplifier, the minimum noise measure can be realized in a stable amplifier.

TABLE I
COMPUTED NOISE PARAMETERS FOR THE FUJITSU FHX01A HEMT

	Common-Emitter	Common-Base	Common-Collector
F_{opt} (min NF)	2.800 (522 deg)*	3.000 (580 deg)	2.758 (510 deg)
R	34.0*	36.9	58.7
Gopt	.01919*	.01890	.01187
Bopt	.01509*	.01278	.01693
Avail Gain	5.416 (7.34 dB)	10.49 (10.2 dB)	1.867 (2.71 dB)
M	2.208 (640.2 deg)	2.21 (641.0 deg)	3.79 (1098 deg)
M_{min}	2.20134 (638 deg)	2.20134 (638 deg)	2.20134 (638 deg)
Gs	.01927	.01732	.00460
Bs	.01668	.01172	.01403
Avail Gain	5.547 (7.44 dB)	11.34 (10.5 dB)	-6.52 (see text)
F	2.804 (523 deg)	3.007 (582 deg)	2.864 (540 deg)

*These data, and the common-emitter S parameters (below), taken from the data sheet for the transistor, were used to calculate the rest of the table:

$$S_{11} = -0.0773 + j0.51119 \qquad S_{12} = 0.03388 - j0.09515$$

$$S_{21} = 0.1840 - j1.4170 \qquad S_{22} = 0.66384 + j0.21569.$$

V. CONCLUSION

Formulas were derived to transform noise parameters when the terminals of a three-terminal amplifier are interchanged. It was shown that the minimum noise measure must be the same for the common-emitter, common-base, and common-collector configurations. A practical example was given to confirm this invariance. High-gain amplifiers with the minimum noise figure can be built with any of the three configurations or combination thereof. The choice of configuration can be (and is) determined by factors such as ease of stabilization or bandwidth.

REFERENCES

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Microwave Shielding Effectiveness of EC-Coated Dielectric Slabs

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***Abstract*—The purpose of this paper is to derive correct formulas for the microwave shielding effectiveness (SE) of a thin metallic layer deposited on top of a dielectric slab. For coatings much thinner than the skin depth, the following holds: (a) In a half-wave geometry, SE is a function of the sheet resistance only, SE (in dB) $= 20 \times \log(1 + 188.5/R_s)$ if R_s is in ohms per square; (b) in a quarter-wave geometry, SE (in dB) $= 20 \times \log[(1+\epsilon_r)/(2\sqrt{\epsilon_r}) + 188.5/(\sqrt{\epsilon_r}\, R_s)]$, where ϵ_r refers to the dielectric constant of the substrate. These formulas provide upper and lower limits for the effective shielding performance of an electroconductively coated dielectric slab.**

I. INTRODUCTION

Thin metallic films or stacks deposited upon glass substrates are known to attenuate incident radio-frequency radiation and, therefore, can be used to protect sensitive components against electromagnetic interference effects. At microwave frequencies, the case of interest is that of a uniform plane wave normally incident upon a "thin" electroconductive (EC) layer backed by a "thick" dielectric slab as illustrated in Fig. 1. The two relevant papers that have appeared in this TRANSACTIONS [1], [2] do not constitute a satisfactory treatment of the shielding effectiveness of such configurations. Liao's formula [1], which rests upon a procedure developed by Lassiter [3] for investigating the near-field situation, is basically incorrect and holds only under very special conditions. The work of Hansen and Pawlewicz [2], on the other hand, applies only to free-standing thin metallic sheets. My purpose here is to present a comprehensive but simple treatment of the microwave attenuation induced by an EC-coated plane-parallel dielectric and, in particular, to provide useful solutions for assessing the shielding effectiveness in an engineering-type environment.

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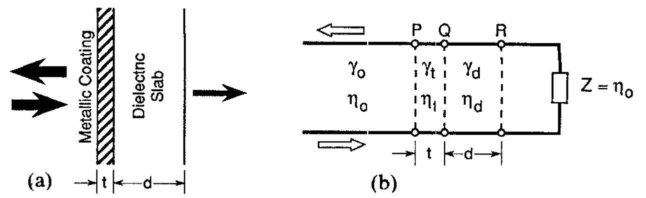

Fig. 1. Electroconductively (EC) coated dielectric slab (a) Electromagnetic shielding results from reflections at impedance discontinuities and absorption in the metal layer. (b) Equivalent transmission-line model including characteristic constants.

The shielding effectiveness (SE) is best defined in terms of the reduction in field intensity [SE (in dB) $= -20 \times \log(E_t/E_i)$] resulting from reflections and losses that occur upon inserting the "barrier" [4]. In the context of conventional transmission-line theory as formulated by Schelkunoff [5], which I will use to describe the propagation of a plane electromagnetic wave through the multilayer structure sketched in Fig. 1(a), the ratio of transmitted to incident electric fields corresponds to the voltage transmission coefficient T_V; the shielding effectiveness (in decibels) is therefore given by

$$SE = 10 \times \log[1/(T_V T_V^*)]. \tag{1}$$

The transmission coefficient T_V can be obtained on the basis of postulating that the metallic layer and the dielectric slab are both equivalent to sections of a transmission line as modeled in Fig. 1(b), that is, inserted into a transmission line of characteristic impedance η_0 terminating in a load impedance $Z = \eta_0$. The discontinuities at points P, Q, and R thus delineate two transmission-line sections of length t and d, each with its own set of characteristic constants. At this point, it is recalled that, in an isotropic medium of permeability μ and permittivity ϵ, the propagation constant of an electromagnetic wave of circular frequency ω is

$$\gamma = \sqrt{i\omega\mu(\sigma + i\omega\epsilon)} \tag{2}$$

where i stands for $\sqrt{-1}$ and σ designates the electrical conductivity. The intrinsic impedance of that medium is

$$\eta = \sqrt{\frac{i\omega\mu}{\sigma + i\omega\epsilon}} \tag{3}$$

which yields $\eta_0 = \sqrt{\mu_0/\epsilon_0} = 377\ \Omega$ for free space.

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